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FULLY IMPLICIT NAVIER-STOKES CODE IN VELOCITY-VORTICITY FORMULATION

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Abstract. We present parallel implementations in PETSc of a fully implicit discretizing in time and space to solve the 3D incompressible Navier-Stokes equations in velocity-vorticity formulation applied to the lid-driven cavity with aspect ratio 3:1

1 INTRODUCTION

From our knowledge there is no implementation of the 3D Navier-Stokes equations in velocity-vorticity formulation fully implicit. Some results exist for a fully implicit formulation of 3D Navier-Stokes equation in velocity-pressure equation [5]. In velocity-vorticity formulation we have 6 equations to solve [1]. We show a PETSc implementation through the SNES PETSc [3, 4] solvers to solve nonlinear systems. Some numerical investigations to enhance the performance of nonlinear solvers will be performed, and some adaptative time stepping to control the error will be investigated.

2 TOTALY IMPLICIT FORMULATION

The totally implicitly discretizing (with Euler) of the 3D Navier-Stokes equations in velocity-vorticity ($\vec{\mathbf{V}} - \vec{\omega}$) formulation writes :

$$\begin{pmatrix} (\mathbb{I} - \frac{\Delta t}{Re}\Delta)(\vec{\omega}^{n+1} - \Delta t \vec{\nabla} \times (\vec{\mathbf{V}}^{n+1} \times \vec{\omega}^{n+1}) - \vec{\omega}^{n} \\ \Delta \vec{\mathbf{V}}^{n+1} + \nabla \times \vec{\omega}^{n+1} \end{pmatrix} = \vec{0} \tag{1}$$

with fully implicit boundary condition. For example the boundary conditions at $z = -\frac{1}{2}$ writes with discretizing $\dfrac{\partial \omega_y}{\partial z}$ and $\dfrac{\partial \omega_x}{\partial z}$ at the first order for the lid-driven cavity problem:

$$\begin{aligned}
\omega_x^{n+1}(x, y, -\frac{1}{2}) + \omega_x^{n+1}(x, y, -\frac{1}{2} + \Delta z) &= -\frac{2}{\Delta z}(V_y^{n+1}(x, y, -\frac{1}{2} + \Delta z) - V_y^{n+1}(x, y, -\frac{1}{2})) \\
\omega_y^{n+1}(x, y, -\frac{1}{2}) + \omega_y^{n+1}(x, y, -\frac{1}{2} + \Delta z) &= \frac{2}{\Delta z}(V_x^{n+1}(x, y, -\frac{1}{2} + \Delta z) - V_x^{n+1}(x, y, -\frac{1}{2})) \\
\omega_z^{n+1}(x, y, -\frac{1}{2}) &= 0
\end{aligned}$$

:

$$
\begin{aligned}
\omega_x^{n+1}(x,y,-\frac{1}{2})+\omega_x^{n+1}(x,y,-\frac{1}{2}+\Delta z) &= -\frac{2}{\Delta z}(V_y^{n+1}(x,y,-\frac{1}{2}+\Delta z)-V_y(x,y,-\frac{1}{2}))\\
\omega_y^{n+1}(x,y,-\frac{1}{2})+\omega_y^{n+1}(x,y,-\frac{1}{2}+\Delta z) &= \frac{2}{\Delta z}(V_x^{n+1}(x,y,-\frac{1}{2}+\Delta z)-V_x^{n+1}(x,y,-\frac{1}{2}))\\
\omega_z^{n+1}(x,y,-\frac{1}{2}) &= 0
\end{aligned}
$$

3 RESULTS

We implement in PETSc this fully implicit strategy to solve the lid driven cavity problem with aspect ration 3:1 and Reynolds 3200. The space discretizing in space is second order finite differences. The implementation consists to define the SNES object representing the 6 discretized equations with their boundary conditions. We used the NGMRES solver implementing the Anderson mixing.

Table 1 represents the parallel efficiency obtained per time steps for the $192 \times 64 \times 64$ mesh on a bulx DLC with processor Xeon E5-2690v3 1x12C 2.6Ghz and 110Gb memory and Infinityband FDR network, from the CINES french national computing ressource.

P	1	2	4	16	32	64	128	256	512	1024	2048
time (s)	137.6	65.8	43.55	13.7	4.54	2.53	1.50	1.00	1.22	0.91	1.18

Table 1: Time per iteration: occigen dt=0.005 192x64x64

Figure 1 gives the flow behavior at time $T = 76$ with the ngmres solver for two time steps $dt = 2.10^{-2}$ and $dt = 5.10^{-3}$. We see a good agreement between the two computations and see the advantage of this implementation that has no CFL condition. The limitation for the time step is only due to physics consideration, in order to catch the right dynamics and we see that we can take a 4 times greater time step for this flow behavior computation.

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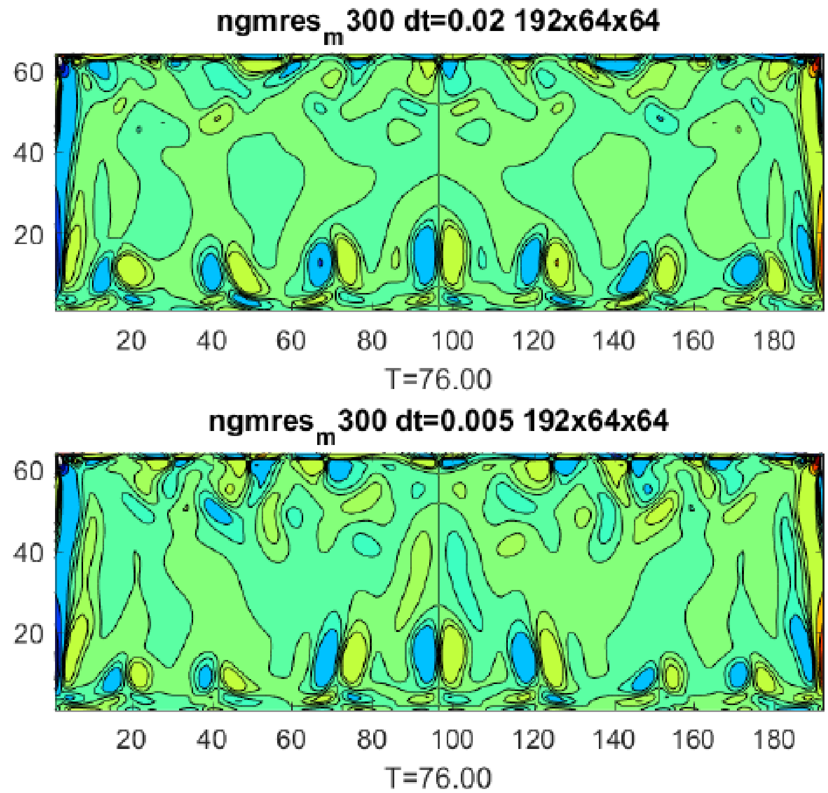

Figure 1: Comparison for two time steps $dt = 2.10^{-2}$ (top) and $dt = 5.10^{-3}$ (bottom).

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