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A PARALLEL MONOLITHIC APPROACH FOR THE INCOMPRESSIBLE MAGNETOHYDRODYNAMICS EQUATIONS

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Abstract. A parallel numerical algorithm has been developed to solve the incompressible magnetohydrodynamics (MHD) equations in a fully coupled form. The numerical methods based on the side-centered unstructured finite volume formulation where the vector variables are defined at face/edge mid-points, meanwhile the scalar quantities are defined at element centroid. The resulting algebraic equations are solved in a monolithic manner using a one-level restricted additive Schwarz preconditioner with a block-incomplete factorization in order to avoid any time step restrictions forced by stability requirements. The implementation of the iterative solver is based on the Portable Extensible Toolkit for Scientific computation (PETSc) library to improve its parallel performance. The numerical algorithm is initially validated by solving the electrically conducting fluid in a rectangular channel corresponding to Shercliff and Hunt analytic solutions. Then the algorithm is used to solve the electrically conducting fluid around a confined square prism in a rectangular channel.

1 INTRODUCTION

Magnetohydrodynamics mainly concerns with the dynamics of magnetic fields in electrically conducting fluids, e.g. in plasmas and liquid metals, and understanding the fluid behavior under the influence of electromagnetic fields plays a crucial role in a large number of applications in science and engineering such as stellar and planetary magnetic fields, solar wind-earth magnetospheric interactions, magnetically confined plasma for fusion energy devices, magnetohydrodynamic generators, liquid-metal blankets, electromagnetic pumps, stirring of liquid metals, hypersonic reentry, magnetohydrodynamic heat shield, electric propulsion, etc. The present study proposes a novel parallel face-centered unstructured finite volume formulation for the solution of the incompressible MHD equations in a fully coupled form, where the velocity and magnetic filed vector components are defined at the center of edges/faces, meanwhile the pressure term is defined at the element centroid. The resulting system of algebraic equations is solved in a monolithic manner. The main advantage of the monolithic approaches is their robustness, but they require the solution of large systems of coupled non-linear equations with effective iterative solvers and appropriate preconditioning techniques. In the present paper, the original system of equations is multiplied with an upper triangular right preconditioner which results in a scaled discrete Laplacian instead of zero blocks in the original system due to the divergence-free constraints. Then a one-level restricted additive Schwarz preconditioner with a block-incomplete factorization within each partitioned sub-domains is utilized for the modified fully coupled system. The implementation of the preconditioned Krylov subspace algorithm, matrix-matrix multiplication and the restricted additive Schwarz preconditioner are carried out using the Portable Extensible Toolkit for Scientific computation (PETSc) software package developed at the Argonne National Laboratories.

2 MATHEMATICAL AND NUMERICAL FORMULATION

The non-dimensional equations governing the incompressible resistive magnetohydrodynamics (MHD) are given as a coupling between the incompressible Navier-Stokes equation and the incompressible induction equation in the following integral form:

$$Re\iiint_{\Omega_d}\frac{\partial \mathbf{u}}{\partial t}dV + Re\oint\!\!\oint_{\partial\Omega_d}[\mathbf{n}\cdot\mathbf{u}]\mathbf{u}dS - ReS\oint\!\!\oint_{\partial\Omega_d}[\mathbf{n}\cdot\mathbf{B}]\mathbf{B}dS + Re\oint\!\!\oint_{\partial\Omega_d}\mathbf{n}PdS = \oint\!\!\oint_{\partial\Omega_d}\mathbf{n}\cdot\nabla\mathbf{u}dS \quad (1)$$

$$Re_m\iiint_{\Omega_d}\frac{\partial \mathbf{B}}{\partial t}dV + Re_m\oint\!\!\oint_{\partial\Omega_d}[\mathbf{n}\cdot\mathbf{u}]\mathbf{B}dS - Re_m\oint\!\!\oint_{\partial\Omega_d}[\mathbf{n}\cdot\mathbf{B}]\mathbf{u}dS + Re_m\oint\!\!\oint_{\partial\Omega_d}\mathbf{n}qdS = \oint\!\!\oint_{\partial\Omega_d}\mathbf{n}\cdot\nabla\mathbf{B}dS \quad (2)$$

$$-\oint\!\!\oint_{\partial\Omega_e}\mathbf{n}\cdot\mathbf{u}dS = 0 \quad (3)$$

$$-\oint\!\!\oint_{\partial\Omega_e}\mathbf{n}\cdot\mathbf{B}dS = 0 \quad (4)$$

where $\mathbf{u}$ is the fluid velocity, $\mathbf{B}$ is the magnetic field, ρ is the fluid density, μ_m is the magnetic permeability and μ is the dynamic viscosity of the fluid. The total pressure P is given as $P = p + \frac{1}{2}S\left\|\mathbf{B}\right\|^2$. The non-dimensional numbers are the Reynolds number Re, the magnetic Reynolds number Re_m and the coupling number S

$$Re = \frac{\rho U L}{\mu}, \quad Re_m = \mu_m\sigma U L, \quad S = \frac{B^2}{\rho\mu_m U^2}$$

These numbers are also related to the Stuart (magnetic interaction) number $N = SRe_m$ and the Hartmann number $Ha = \sqrt{SReRe_m}$. In order to impose the solenoidal property of magnetic field, the gradient of a Lagrange multiplier q introduced to the magnetic induction equation as proposed in [1]. In the present study, the face-centered finite volume method [2] has been extended for the solution of the incompressible magnetohydrodynamics equations. The local velocity vector components (u, v, w) and magnetic filed vector components (B_x, B_y, B_z) are defined at the mid-point of each hexahedral element face, meanwhile the pressure and Lagrange multiplier are defined at element centroid. Hence, the discretizations of the momentum equation and the magnetic induction equation are done over the dual control volume, meanwhile the continuity equation and Gauss' law for magnetism are discretized over the element itself. The resulting system of algebraic equations is solved in a monolithic manner in order to avoid any time step restrictions forced by stability requirements. Hence, the restricted additive Schwarz preconditioner combined with the FGMRES(m) Krylov subspace algorithm has been employed to solve the fully coupled system using the Portable Extensible Toolkit for Scientific computation (PETSc) library [3].

3 NUMERICAL RESULTS

The numerical algorithm is initially validated with the analytical solutions of electrically conducting fluid in a rectangular channel corresponding to Shercliff [4] and Hunt [5] solutions. The computational domain is set to $[-5,25]\times[-1,1]\times[-2,2]$. The externally applied magnetic field is given by $\mathbf{B} = (0,1,0)^\top$. The analytical solution is imposed at the inlet and the traction free boundary condition is used at the outlet. The calculation is carried out at $Re = 100$, $Re_m = 1$ and $S = 1$ ($Ha = 10$). The nondimensional numbers are based on the channel half height and the average mass flow rate. The numerical calculation is provided in Figure 1-a with the velocity profiles at the $z = 0$ center plane. The numerical calculation indicates the formation of the Hartmann layer on the solid walls, which is consistent with the analytical solution. Then a square of $[-0.5,0.5]\times[-0.5,0.5]\times[-0.5,0.5]$ prism is inserted into the channel and the electrically insulating boundary condition with no-slip velocity is imposed on the prism. The modified flow structure is presented in Figure 1-b corresponding to the $z = 0$ mid-plane. As the Hartmann number increases the separation bubble behind the square prism is significantly reduced and the pressure gradient in the channel is increased. The flow converges to the Shercliff solution at the outlet.

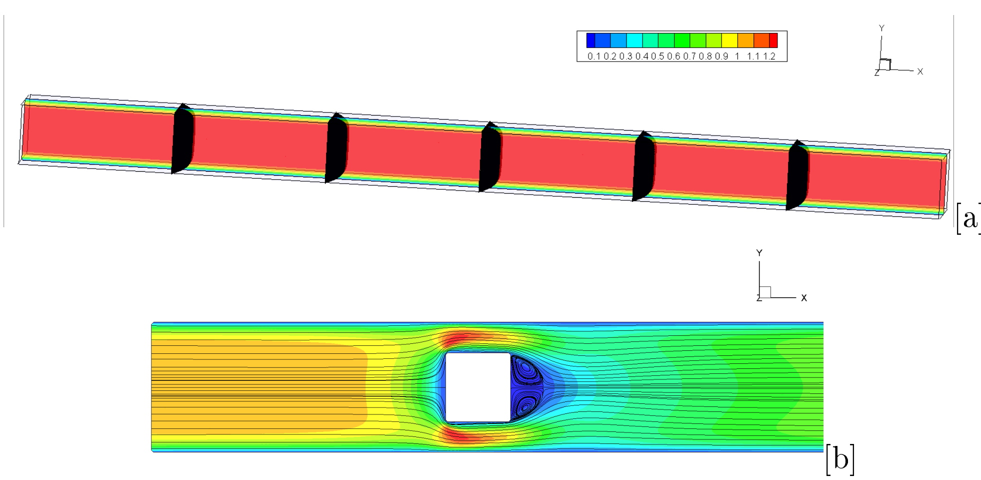

Figure 1: The contours of $x-$velocity at $z = 0$ plane for rectangular channel with insulating walls [a] and channel with unit cube inside [b].

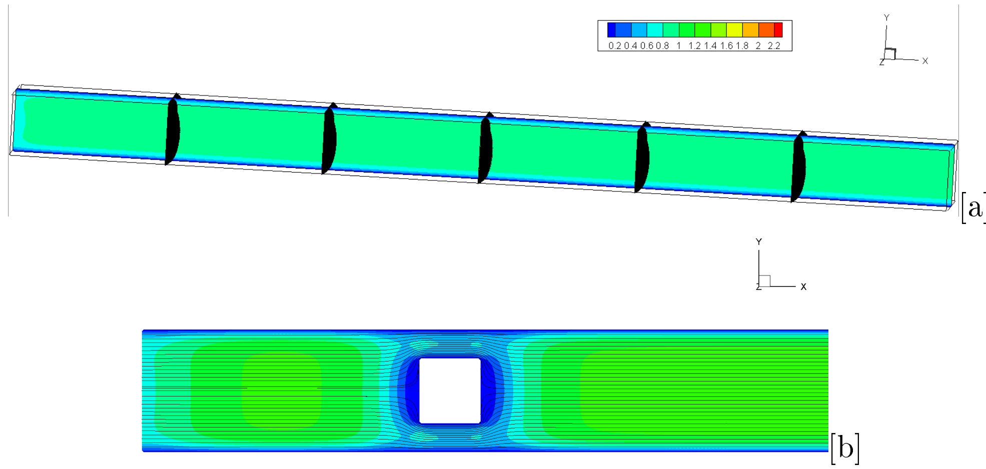

Figure 2: The contours of $x-$velocity at $z = 0$ plane for rectangular channel with conducting Hartmann walls [a] and channel with unit cube inside [b].

The second set of numerical solutions correspond to the perfectly electrically conducting channel walls corresponding to Hunt [5] solution. The computed solution with the same nondimensional numbers is provided in Figure 2-a. The velocity profiles indicates that the velocity increases next to the perfectly insulating side walls creating ”M” type velocity profile in spanwise direction with a relatively low velocity at $z = 0$ mid-plane. Then the cubic prism with the electrically insulating walls is inserted into the channel in order to demonstrate the effect of the perfectly electrically conducting channel walls. The similar flow features are observed for the electrically conducting fluid around a confined square prism as seen in Figure 2-b. However, the separation bubble is diminished at mid-plane due to increased pressure gradient.

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