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PARALLEL DIRECT NUMERICAL SIMULATION AND ANALYSIS OF RAYLEIGH-BÉNARD CONVECTION

I. YILMAZ

Dept. of Mechanical Engineering,
Fac. of Engineering and Natural Sciences,
Istanbul Bilgi University, Eyup, 34060, Istanbul, Turkey
e-mail: ilyas.yilmaz@bilgi.edu.tr

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Abstract. Direct Numerical Simulation of turbulent Rayleigh-Bénard Convection up to Rayleigh number 10^8 is performed using an all-speed, non-dissipative, discrete kinetic energy conserving algorithm and a parallel solver based on it. It is first shown by comparing with the data from literature that the algorithm is capable of reproducing the complex physics of the problem without relaying on any assumption like the Oberbeck-Boussinesq. The turbulent Rayleigh-Bénard Convection is then analyzed in detail using low- and high-order turbulent statistics and various other diagnostics derived. Additionally, Nusselt-Rayleigh scaling properties are studied and a scaling relation is provided.

1 INTRODUCTION

During the years following Lord Rayleigh's theoretical analysis to explain Bénard's observations made through his experiments on the evolution of flow between two horizontal plates with different temperatures, this setup of thermal convection problem gained too much attention and was named Rayleigh-Bénard Convection (RBC) in time. It has been extensively studied both experimentally and numerically by many researchers.

Numerical simulations have mostly been performed using the incompressible Navier-Stokes equations with the Oberbeck-Boussinesq (OB) assumption which neglects changes occur in the density due to temperature difference except in the buoyancy term in momentum equation. The dynamic viscosity is also assumed as constant and independent from the temperature. For small temperature differences, the OB assumption simplifies the analysis and works well. However, with an increasing temperature difference which is very common situation for flows in nature and engineering as well, the assumption gradually looses its validity and may not represent the physics of flows accurately. Thus, an increasing interest to thermal convection flows with relatively larger temperature differences can recently be observed in both scientific and engineering communities. Fire modeling, nuclear reactor flows, combustion and astrophysical flows are the typical examples of thermal convection with large temperature differences where low to moderate compressibility effects are observed and must be taken into account for accurate modeling.

In order to overcome this limitation, the low-Mach number approximation with variable density and dynamic viscosity can be considered as a promising way. Although it has many advantages such as allowing to study large temperature differences which corresponds to moderate to high Rayleigh numbers and better representation of flow physics, classical versions of this approach which include preconditioning to the governing equations come with an increasing numerical cost and computational complexity that prevent wide-spread use of low-Mach number approximation. Recently, efficient, non-preconditioned, robust at high Reynolds numbers and easy-to-parallelize versions were proposed and verified successfully for various flows that can also be applicable for analyzing heat transfer problems in transitional/turbulent regimes. Since, to the best knowledge of the author, not too much examples of efficient, low-to-moderate Mach number algorithms applied to turbulent thermal convection flows are available in the literature, it is also worth to perform such studies to check their performances and understand the changes in the flows physics with variable properties as well.

The aim of this study is twofold. First, to show that low-Mach number, variable-density approach is able to capture the evolution of complex flow field and the heat transfer characteristics of RBC correctly, without relying on any assumption, via comparisons with the previous data available in the literature. Then, to analyze RBC in detail using various turbulent statistics and other flow diagnostics in order to understand the scaling behavior of the turbulent convection at different Rayleigh numbers which correspond to *soft* and *hard* turbulent regimes.

For this purpose, a fully-implicit, fully-parallel, efficient DNS solver [1] based on an advanced, all-speed algorithm [2] is used to simulate the RBC and perform the further analysis.

2 NUMERICAL METHODOLOGY

A fully-implicit, all-speed, non-dissipative, discrete kinetic energy conserving algorithm that solves the set of time-dependent, low-Mach number scaled, three-dimensional compressible Navier-Stokes equations is used. A gravitational source term is also added to these equations. It is also second-order, co-located, *pressure-correction* type, iterative *predictor-corrector* algorithm. Note that, since the approach followed here allows not only the density but also the dynamic viscosity to vary during the simulation, it is physically more consistent with large temperature differences.

The algorithm was also modified to enhance the convergence properties by preventing possible oscillations for successive pressure corrections in time at low-Mach numbers [1].

3 SOLVER DETAILS

An in-house, fully-parallel, three-dimensional DNS code which was previously developed based on the algorithm mentioned above and the PETSc parallel library [3] is used in the simulations. It is a single-block, structured, fully-implicit, finite-volume solver works on uniform/non-uniform Cartesian grids. It was successfully applied to transitional and turbulent flows with buoyancy [1]. The linear systems stemming from the fully-implicit discretization are efficiently solved by incomplete LU-preconditioned GMRES. During the parallel performance tests, very good speed-up and efficiency results were obtained [1].

4 SIMULATION SETUP

RBC occurs when a horizontal layer of fluid between plates is heated from below and cooled from above. It is an example of thermally- and buoyancy-driven instability. Flow regimes in RBC are characterized by the Rayleigh number defined as $Ra = \frac{\rho g \beta \Delta T H^3}{\mu \kappa}$ where $g, \beta, \Delta T, H$ and κ are the gravity, thermal expansion coefficient, temperature difference, domain height and thermal diffusivity respectively.

Three different Rayleigh numbers, $Ra = 6.3 \times 10^5, 10^7$ and 10^8, ranging from *soft* to *hard* turbulent regimes are studied. The simulations are performed on $4:1:4$ aspect ratio domain. The Kolmogorov length-scale can be estimated via $\eta/H_{Kol} = \sqrt{Pr}/[(Nu-1)Ra]^{\frac{1}{4}}$ using the $Nu - Ra$ scaling given by Kerr [4]. The non-dimensional vertical grid spacings perfectly satisfies this estimation and also resolve the thermal boundary layers, which can be estimated as $\lambda_T = 1/2Nu$, using at least $5-6$ cells for each Ra number. The horizontal grid resolution is computed using Grötzbach's criterion, $\eta/H_{Gro} \leq \pi\eta/H_{Kol}$. Pr is the Prandtl number set to 0.71. $u_c = \sqrt{g\beta\Delta T H}$ is the convective velocity and $\tau_c = H/u_c$ is the convective time-scale. The reference Mach number, M_r, is set to 0.01. The flow is followed up to $2100\tau_c$ and the statistics are collected during the period of last $700\tau_c$. For all runs, ΔT is set to $30\,K$.

Periodic boundary conditions are used for the stream-wise and the span-wise directions for all variables. Top and bottom surfaces are no-sip walls with constant temperatures. The homogeneous Neumann boundary condition is applied for the pressure at walls.

The flow is initialized with zero velocity field. The initial temperature, density and pressure distributions are $T = T_0 - (\frac{\Delta T}{H})y$, $\rho = \rho_0(1 + \beta(\frac{\Delta T}{H})y)$ and $p = p_0 - \rho g y$ respectively. Table 1 and 2 summarize the case and resolution details respectively.

Table 1: Summary of the simulation details

Ra	u_c	τ_c	Ny	$Nx = Nz$	Δt
6.3×10^5	0.242	0.240	96	64	$0.008\tau_c$
10^7	0.384	0.380	192	170	$0.005\tau_c$
10^8	0.563	0.558	288	384	$0.002\tau_c$

Table 2: Summary of the grid resolution details

Ra	η/H_{Kol}	Δ_y	η/H_{Gro}	$\Delta_x = \Delta_z$	λ_T	$N_{\lambda_T} (\approx \lambda_T/\Delta_y)$
6.3×10^5	0.0188	0.0104	0.0590	0.0625	0.0674	6
10^7	0.0076	0.0052	0.0239	0.0235	0.0314	5
10^8	0.0036	0.0035	0.0114	0.0104	0.0166	5

5 RESULTS AND DISCUSSION

In order to analyze the evolution and the final state of RBC, mean and turbulent statistics (both low- and high-order) of various flow variables are computed. Some further derived quantities such as skewness, strain rate, turbulent heat flux, turbulent kinetic energy and Nusselt number are also computed. Additionally, Nu-Ra scaling properties are investigated and a scaling relation is provided. Previous experimental and DNS data from the literature are used for comparison where available.

The results are in well-agreement with the previous studies. Due to the limited space, only some of the results with $Ra = 6.3 \times 10^5$ are presented here in Fig 1. It is shown that the complex physics of RBC is well reproduced. The detailed analysis of results using the various diagnostics mentioned above will be provided in the full manuscript.

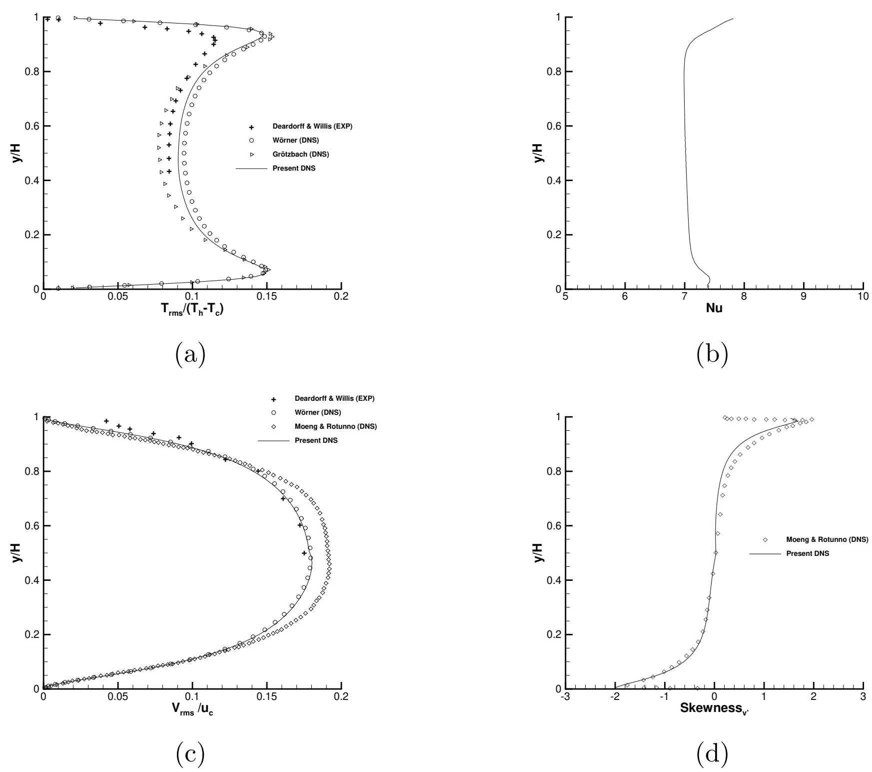

Figure 1: The normal-wise distributions of T_{rms}(a), Nu(b) and V_{rms}(c) and the skewness(d) in comparison with the data available from the previous exp. and DNS studies.

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